EE 369

POWER SYSTEM ANALYSIS

Lecture 8

Per Unit

Tom Overbye and Ross Baldick

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Per Unit Calculations

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

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Per Unit Conversion Procedure, 1ϕ

1. Pick a 1ϕ VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

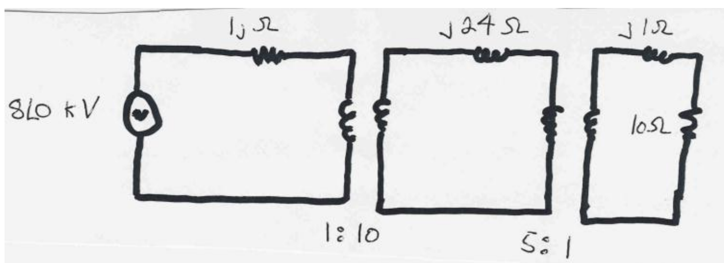
Note, per unit conversion affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

Per Unit Solution Procedure

1. Convert to per unit (p.u.) (many problems are already in per unit)
2. Solve
3. Convert back to actual as necessary

Per Unit Example

Solve for the current, load voltage and load power in the circuit shown below using per unit analysis with an S_B of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV, respectively.



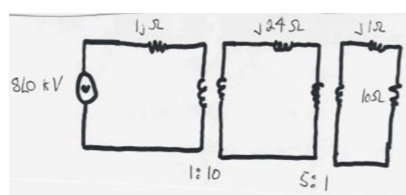
Original Circuit

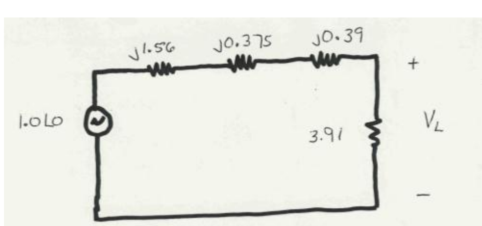
Per Unit Example, cont'd

$$Z_B^{Left} = \frac{8^2 (\text{kV})^2}{100\text{MVA}} = 0.64\Omega$$

$$Z_B^{Middle} = \frac{80^2 (\text{kV})^2}{100\text{MVA}} = 64\Omega$$

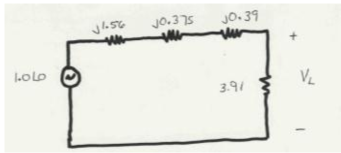
$$Z_B^{Right} = \frac{16^2 (\text{kV})^2}{100\text{MVA}} = 2.56\Omega$$





Same circuit, with values expressed in per unit.

Per Unit Example, cont'd



$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Per Unit Example, cont'd

To convert back to actual values just multiply the per unit values by their per unit base

$$V_L^{\text{Actual}} = 0.859\angle -30.8^\circ \times 16 \text{ kV} = 13.7\angle -30.8^\circ \text{ kV}$$

$$S_L^{\text{Actual}} = 0.189\angle 0^\circ \times 100 \text{ MVA} = 18.9\angle 0^\circ \text{ MVA}$$

$$S_G^{\text{Actual}} = 0.22\angle 30.8^\circ \times 100 \text{ MVA} = 22.0\angle 30.8^\circ \text{ MVA}$$

$$I_B^{\text{Middle}} = \frac{100 \text{ MVA}}{80 \text{ kV}} = 1250 \text{ Amps}$$

$$I_{\text{Middle}}^{\text{Actual}} = 0.22\angle -30.8^\circ \times 1250 = 275\angle -30.8^\circ \text{ A}$$

Three Phase Per Unit

Procedure is very similar to 1ϕ except we use a 3ϕ VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system, $S_B^{3\phi}$
2. Pick a voltage base for each different voltage level, $V_{B,LL}$. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase using the corresponding single phase VA base and voltage base!

Three Phase Per Unit, cont'd

4. Calculate the current base, I_B

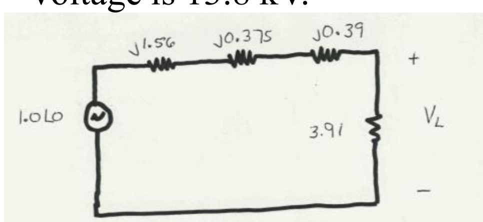
$$I_{\mathrm{B}}^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_{\mathrm{B}}^{1\phi}$$

Exactly the same current bases as with single phase!

5. Convert actual values to per unit

Three Phase Per Unit Example

- Solve for the current, load voltage and load power in the previous circuit, assuming:
  - –a 3ϕ power base of 300 MVA,
  - –and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages)
  - –the generator is Y-connected so its line to line voltage is 13.8 kV.



Convert to per unit as before.
Note the system is exactly the same!

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3ϕ Per Unit Example, cont'd

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Again, analysis is exactly the same!

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3ϕ Per Unit Example, cont'd

Differences appear when we convert back to actual values

$$V_{L}^{Actual} = 0.859\angle -30.8° \times 27.6 \text{ kV} = 23.8\angle -30.8° \text{ kV}$$

$$S_{L}^{Actual} = 0.189\angle 0° \times 300 \text{ MVA} = 56.7\angle 0° \text{ MVA}$$

$$S_{G}^{Actual} = 0.22\angle 30.8° \times 300 \text{ MVA} = 66.0\angle 30.8° \text{ MVA}$$

$$I_{B}^{Middle} = \frac{300 \text{ MVA}}{\sqrt{3}\, 138 \text{ kV}} = 1250 \text{ Amps}$$ (same current!)

$$I_{Middle}^{Actual} = 0.22\angle -30.8° \times 1250 \text{ Amps} = 275\angle -30.8° \text{ A}$$

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